5. Linear Equations in Two Variables

- **Linear equation in two variables:**

  An equation of the form, $ax + by + c = 0$, where a, b and c are constants, such that a and b are both not zero and x and y are variables is called a linear equation in two variables.
  For example, $2x + 3y + 10 = 0$, $3x + 7y = 0$

- **Elimination Method to Solve a Pair of Linear Equations**

  Example:
  Solve the following pair of linear equations by elimination method.
  $7x - 2y = 10$
  $5x + 3y = 6$

  Solution:
  $7x - 2y = 10$ … (1)
  $5x + 3y = 6$ … (2)
  Multiplying equation (1) by 5 and equation (2) by 7, we get
  $35x - 10y = 50$ … (3)
  $35x + 21y = 42$ … (4)
  Subtracting equation (4) from (3), we get
  $-31y = 8 \Rightarrow y = -\frac{8}{31}$
  Now, using equation (1):

  $7x = 10 + 2y$
  $\Rightarrow x = \frac{1}{7}\left\{10 + 2 \times \frac{-8}{31}\right\} = \frac{42}{31}$.
  Required solution is $\left(\frac{42}{31}, -\frac{8}{31}\right)$.

- **Substitution Method of Solving Pairs of Linear Equations**

  In this method, we have **substituted** the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why this method is known as the **substitution method.**

  Example:
  Solve the following system of equations by substitution method.
  $x - 4y + 7 = 0$
  $3x + 2y = 0$

  Solution:
  The given equations are
  $x - 4y + 7 = 0$ … (1)
  $3x + 2y = 0$ … (2)
  From equation (2),

$3x = -2$

$\Rightarrow x = -\frac{2}{3}y$

Put $x = -\frac{2}{3}y$ in equation (1)

$-\frac{2}{3}y - 4y + 7 = 0$

$\Rightarrow \frac{-2y - 12y}{3} = -7$

$\Rightarrow -14y = -21$

$\Rightarrow y = \frac{-21}{-14} = \frac{3}{2}$

$\therefore x = -\frac{2}{3}\left(\frac{3}{2}\right) = -1$

Therefore, the required solution is $\left(-1, \frac{3}{2}\right)$.